MSc Computer Games and Entertainment

Maths & Graphics Unit 2011/12

Lecturer: Gareth Edwards

First - For Complete Clarity

Course Prerequisites

Course Prerequisites:

- As per Lecture 01, the fundamentals of:
  - C++
  - DirectX
  - 3D Pipeline
    - Application
    - Geometry
    - Rasterizer

- Next week an opportunity to go over the fundamentals AGAIN!

Next Week

- **Wednesday 17th:**
  - From 11:00 to 13:00:
    - Fundamentals
  - From 13:30 to 17:30:
    - Introduction to Houdini
    - Programming for Vectors using C++ Operator Overloading
    - Introduction to Matrices

- **Thursday 18th:**
  - From 13:30 to 15:30 (and beyond?):
    - Houdini lecture by SideFX

Second – A Recap

Last week – Lecture 01: Mathematics for Computer Games

- This Course on Mathematics for Computer Games covers a wide range of topics; from elementary subjects such as Cartesian coordinates systems and subjects such as the dot and cross product, through vectors, matrices, transforms, parametric, curves, to simulation, global illumination model, programmable shaders, and real and non-real time rendering techniques.

- The objective of this course if to ensure that all Students have a broad understanding of the Mathematics required to create and render a 3D Scene.

- For those Students who are already familiar with much of the subject matter, this course might be considered as a refresher; for others it should be a "Call to Arms"; an opportunity to study and understand those parts of the subject matter covered of which they are unsure or unfamiliar.

Course Context – The 3D Pipeline

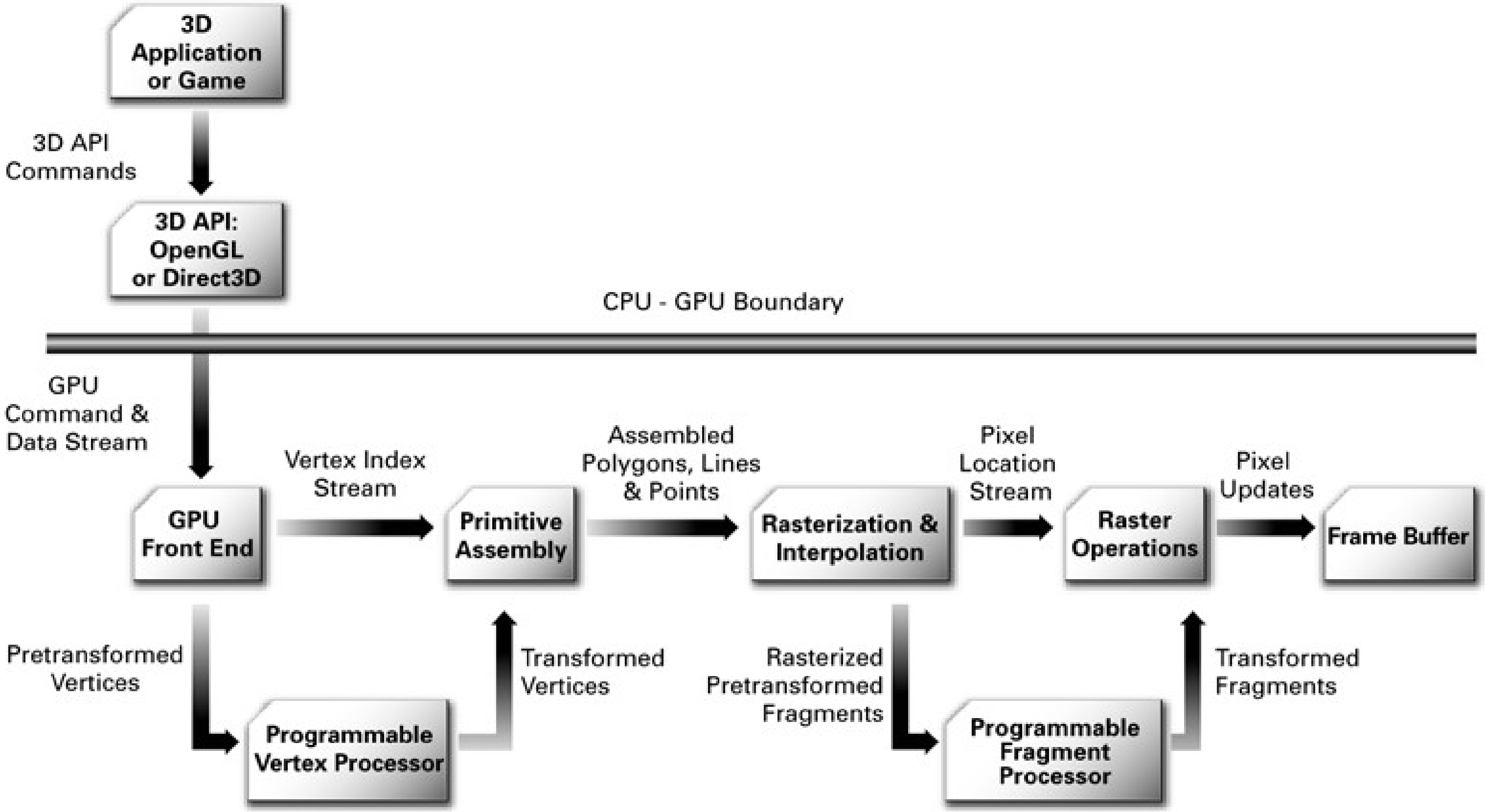
The pipeline is the “engine” that creates images from a description of a 3D scene.

There are 3 conceptual stages of the pipeline:

1. **Application** – executed on the CPU, including creating and then sending graphic primitives to be stored on specialised hardware - the GPU.
2. **Geometry** – from within an application perform “geometrical” operations, which are executed on the GPU, on the graphic primitives stored on the GPU.
3. **Rasterizer** – from within an application render a 3D scene on the GPU.

Cartesian coordinate plane with four points.

Course Context – The 3D Pipeline



Graphic Primitives

The 3D Pipeline uses the GPU to render 3D scenes in real time.

3D scenes are made up of objects.

These objects are constructed from geometry, where:

- Geometry is either pre-created using an interactive modeling package or created procedurally within an application

So this week- Lecture 02 is:

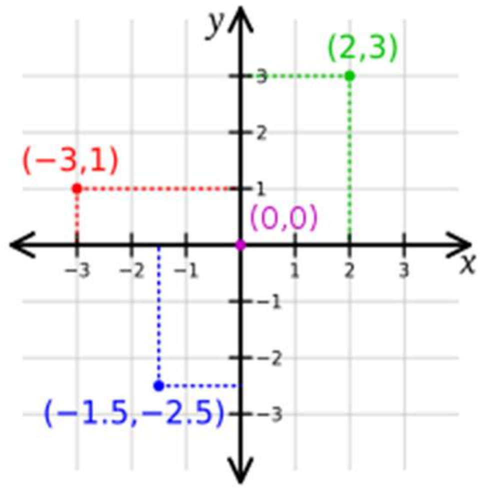
Geometry for
Computer Graphics

Euclidean Geometry

- A mathematical system attributed to the Alexandrian Greek mathematician Euclid, whose ***Elements*** is the earliest known systematic discussion of geometry.
- Euclid's method consists in assuming a small set of intuitively appealing axioms, and deducing many other propositions (theorems) from these.
- Although many of Euclid's results had been stated by earlier mathematicians, Euclid was the first to show how these propositions could fit into a comprehensive deductive and logical system.
- The *Elements* begins with plane geometry, still taught in as the first axiomatic system and the first examples of formal proof.
- It goes on to the solid geometry of three dimensions.
- Today, many other self-consistent non-Euclidean geometries are known.

Cartesian coordinate system

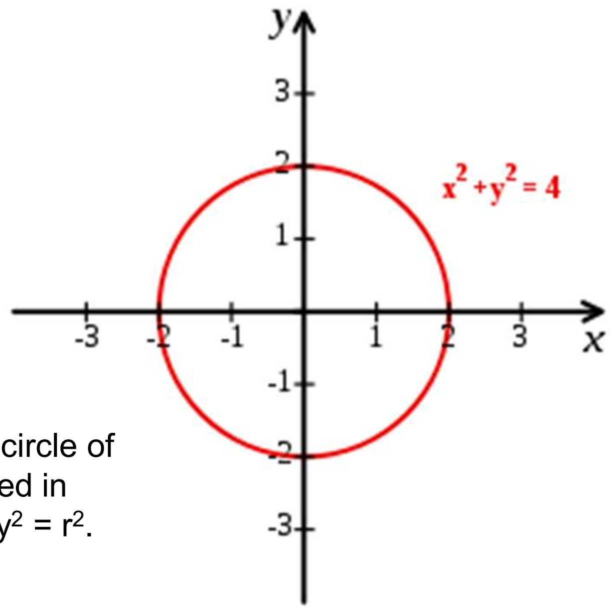
- A **Cartesian coordinate system** specifies each point uniquely in a plane by a pair of numerical **coordinates**, which are the signed distances from the point to two fixed perpendicular directed lines, measured in the same unit of length.
- The invention of Cartesian coordinates in the 17th century by René Descartes revolutionized mathematics by providing the first systematic link between Euclidean geometry and algebra.



Cartesian coordinate plane with four points.

Cartesian coordinate system

- Using the Cartesian coordinate system, geometric shapes (such as curves) can be described by **Cartesian equations**: algebraic equations involving the coordinates of the points lying on the shape.
- For example, a circle of radius 2 may be described as the set of all points whose coordinates *x* and *y* satisfy the equation $x^2 + y^2 = 2^2$.



Cartesian coordinate system with a circle of radius 2 cantered at the origin marked in red. The equation of a circle is $x^2 + y^2 = r^2$.

Point (1)

- A precise location or place on a plane.
  - Usually represented by a dot.
  - A **point** is an exact position or location on a **plane** surface.
  - It is important to understand that a **point** is not a *thing*, but a *place*.
  - We indicate the position of a **point** by placing a dot with a pencil.
  - This dot may have a diameter of, say, 0.2mm, but a **point** has no size.
  - No matter how far you zoomed in, it would still have no width.
  - Since a **point** is a place, not a thing, it has no dimensions.
  - **Points** are usually named by using an upper-case single letter.

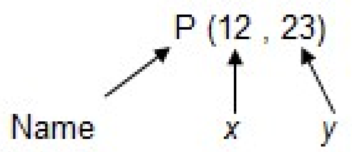
- Note:
  - If a set of **points** all lie in a straight line, they are called 'collinear'.
  - If a set of **points** all lie on the same plane, they are called 'coplanar'.

Point (2)

- **Coordinates of a point**
  - From Latin: coordinare "to set in order, arrange"
  - A pair of numbers defining the position of a **point** on a two-dimensional plane

Point (3)

- **Ordered pair**
  - The coordinates are written as an "ordered pair" as shown below.



  - The letter **P** is simply the name of the **point** and is used to distinguish it from others.
  - The two numbers in parentheses are the x and y coordinates of the **point**.
  - It is called an ordered pair because the order of the two numbers matters - the first is <u>*always*</u> the **x** (horizontal) coordinate. The second is <u>*always*</u> the **y** (vertical) coordinate.
  - The sign of the coordinate is important. A positive number means to go to the right (x) or up(y). Negative numbers mean to go left (x) or down (y).

Point (4)

- **Abscissa**
  - The abscissa is another name for the ***x*** (horizontal) coordinate of a **point**.
  - Pronounced "ab-SISS-ah" (the 'c' is silent).
  - Not used very much.
  - Most commonly, the term "**x-coordinate**" is used.

Point (5)

- **Ordinate**
  - The ordinate is another name for the ***y*** (vertical) coordinate of a **point**.
  - Pronounced "ORD-inet".
  - Not used very much.
  - Most commonly, the term "**y-coordinate**" is used.

Coordinate Geometry

- In coordinate geometry, **points** are located on the **plane** using their coordinates - two numbers that show where the **point** is positioned.

- To achieve this, the **plane** is thought to have two scales at right angles.

- Using a pair of numbers, any **point** on the **plane** can be uniquely described.

Coordinate Plane (1)

- A two-dimensional surface on which **points** are plotted and located by their **x** and **y** coordinates

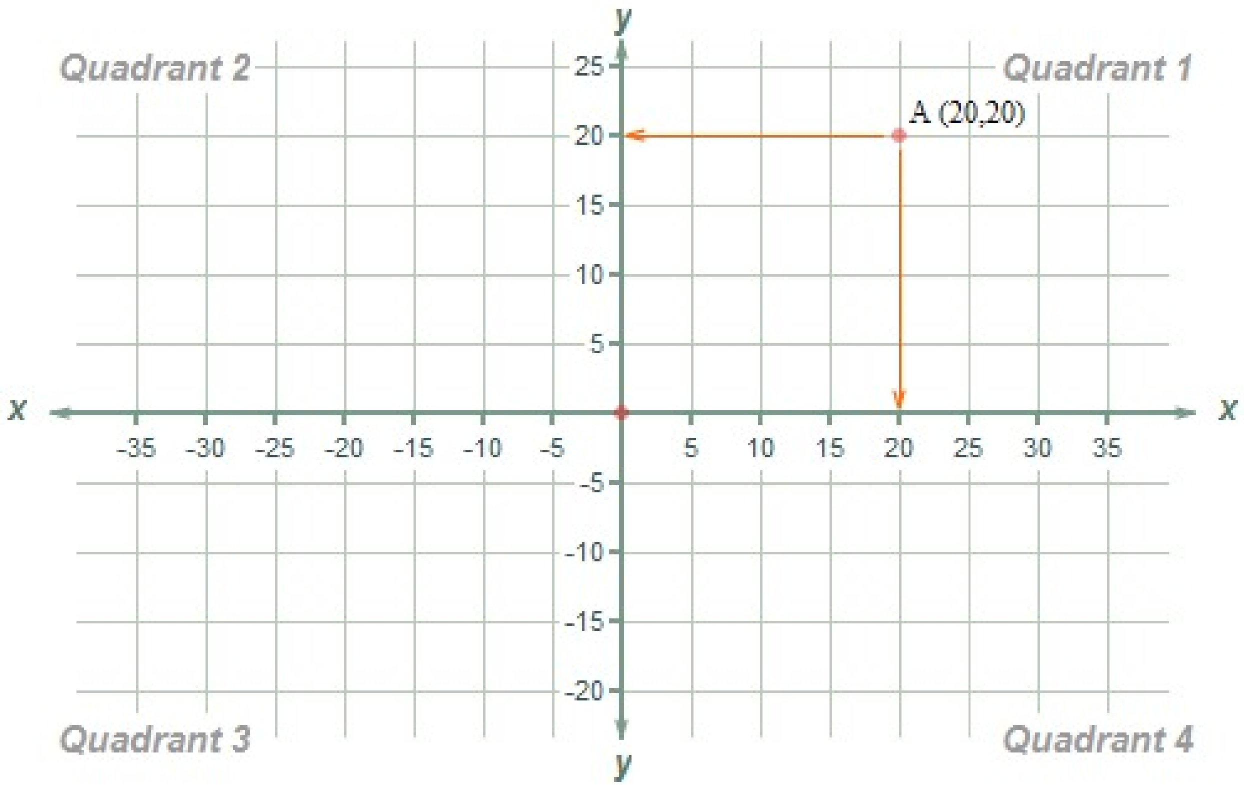
- The **coordinate plane** is a two-dimensional surface on which we can plot **points**, lines and curves. It has two scales, called the ***x*-axis** and ***y*-axis**, at right angles to each other.

- The plural of axis is 'axes' (pronounced "AXE-ease").

- **Points** on the **plane** are located using two numbers - the **x** and **y** coordinates.

- These are the horizontal and vertical distances of the **point** from a specific location called the **origin**.

Coordinate Plane (2)



Coordinate Plane (3)

- ***X* axis**
  - The horizontal scale is called the ***x*-axis**. As you go to the right on the scale from zero, the values are positive and get larger. As you go to the left from zero, they get more and more negative.

- ***Y* axis**
  - The vertical scale is called the ***y*-axis**. As you go up from zero the numbers are increasing in a positive direction. As you go down from zero they get more and more negative.

- **Axis labelling**
  - Along each axis you will see small tic marks with numbers.
  - These are labels to help judge the scale. They are shown every 5 units in the figure above, but can be any increment, and need not be the same on both axes.

Coordinate Plane (4)

- **Origin**
  - The **point** where the two axes cross (at zero on both scales) is called the **origin**.
  - The **origin** is the **point** from which all distances along the **x** and **y axes** are measured. In the figure above you can drag the origin point to reposition it to a more suitable location at any time.

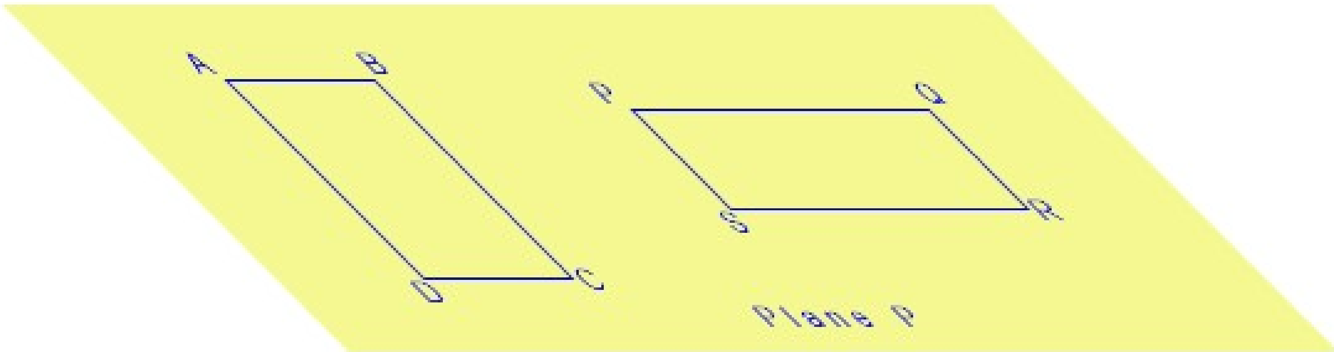
- **Quadrants**
  - The two axes divide the **plane** into four areas called quadrants. The first quadrant, by convention, is the top right, and then they go around counter-clockwise. In the diagram above they are labelled Quadrant 1,2 etc. It is conventional to label them with numerals but we talk about them as "first, second, third, and fourth quadrant". They are also sometimes labelled with Roman numerals: I, II, III and IV.

Collinear points

- From Latin: co- "together", -linearis "belonging to a line,"
- A set of **points** that lie in a straight line
- Obviously two **points** are always **collinear**, since a straight line can always be drawn through two **points**.
- Sometimes it is spelled 'colinear' (with one L).
- We say that "point Q is **collinear** with points P, R and S".
- Or put another way, "the points P, Q, R and S are **collinear**".

Coplanar

- Lying in the same **plane.**
- Two objects are **coplanar** if they both lie in the same **plane**.



- In the figure above, the two rectangles ABCD and PQRS are both **coplanar**.

Plane (1)

- From Latin: plantum - "flat surface,“.

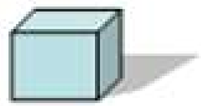
- A flat surface that is infinitely large and with zero thickness.

- Clearly, when you read the above definition, such a thing cannot possibly really exist.

- Imagine a flat sheet of metal. Now make it infinitely large in both directions. This means that no matter how far you go, you never reach its edges. Now imagine that it is so thin that it actually has no thickness at all. In spite of this, it remains completely rigid and flat. This is the **'plane'** in geometry.

Plane (2)

- It fits into a scheme that starts with a **point**, which has no dimensions and goes up through solids which have three dimensions:

point	**line**	**Plane**	**Solid**
Zero dimensions	One dimension	Two dimensions	Three dimensions
			

Plane (3)

- It is difficult to draw **planes**, since the edges have to be drawn. When you see a picture that represents a **plane**, always remember that it actually has no edges, and it is infinitely large.

- The **plane** has two dimensions: length and width. But since the plane is infinitely large, the length and width cannot be measured.

- Just as a **line** is defined by two **points**, a **plane** is defined by three **points**. Given three **points** that are not collinear**,** there is just one **plane** that contains all three.

Parallel planes

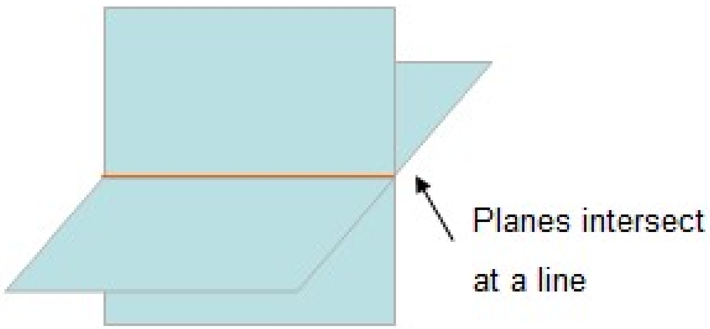
- You can think of **parallel planes** as sheets of cardboard one above the other with a gap between them. **Parallel planes** are the same distance apart everywhere, and so they never touch.

Two parallel planes

Intersecting planes

- If two **planes** are not **parallel**, then they will intersect (cross over) each other somewhere.
- Two **planes** always **intersect** at a **line**, as shown above.

  This is similar to the way two **lines intersect** at a **point**.



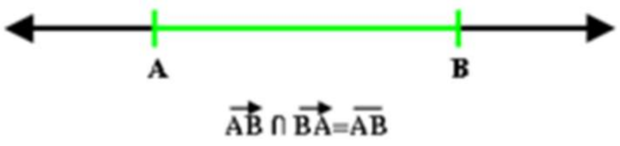
Line segment

- A **line segment** is part of a line that is bounded by two end **points**, and contains every **point** on the **line** between its end **points**.

- Examples of **line segments** include the sides of a triangle or square.

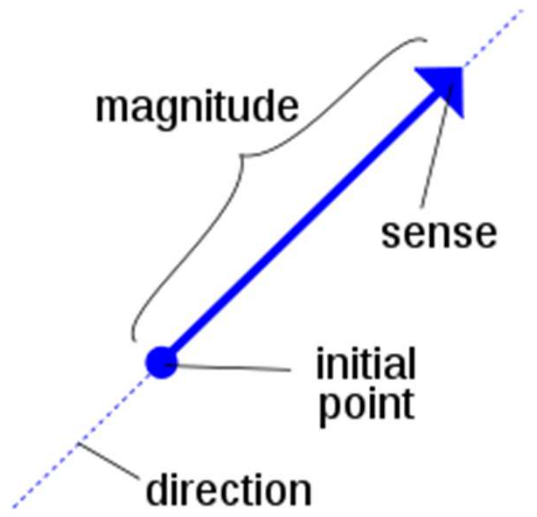
- More generally, when the end **points** are both vertices of a polygon, the **line segment** is either an edge (of that polygon) if they are adjacent vertices, or otherwise a diagonal.

- When the end **points** both lie on a curve such as a circle, a **line segment** is called a chord (of that curve).



Vector (1)

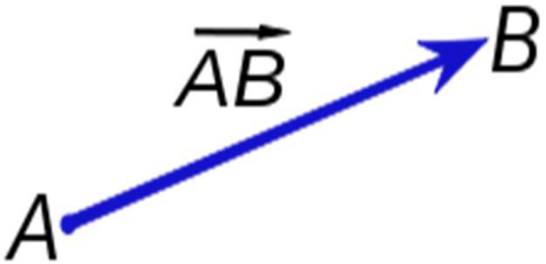
- A **vector** is a geometric object that has either a magnitude (or length) and direction.



Vector (2)

- A **vector** is frequently represented by a **line segment** with a definite direction, or graphically as an arrow, connecting an initial **point** *A* with a terminal **point** *B.*

- This might usually be denoted by $\overrightarrow{AB}$.



Vector (3)

- In three dimensional space **vectors** are identified with triples of numbers corresponding to the **Cartesian coordinates** of the end **point** (*a*,*b*,*c*):

$$\mathbf{a} = (a, b, c).$$

- These numbers are often arranged into a column **vector** or row **vector**, particularly when dealing with matrices, as follows:

$$\mathbf{a} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad \mathbf{a} = [a\ b\ c].$$

Vector - Basic Properties

- The following section uses the **Cartesian coordinate** system with basis **vectors:**

$$\mathbf{e}_1 = (1,0,0),\ \mathbf{e}_2 = (0,1,0),\ \mathbf{e}_3 = (0,0,1)$$

- Assumes that all **vectors** have the origin as a common base **point**.

- A **vector a** will: be written as

$$\mathbf{a} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3.$$

Vector Equality

- Two **vectors** are said to be equal if they have the same magnitude and direction. Equivalently they will be equal if their coordinates are equal.

- So two **vectors**:

$$\mathbf{a} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3$$

- And,

$$\mathbf{b} = b_1\mathbf{e}_1 + b_2\mathbf{e}_2 + b_3\mathbf{e}_3$$

- Are equal if:

$$a_1 = b_1, \quad a_2 = b_2, \quad a_3 = b_3.$$

Vector Addition and Subtraction (1)

- Assume now that **a** and **b** are not necessarily equal **vectors**, but that they may have different magnitudes and directions.

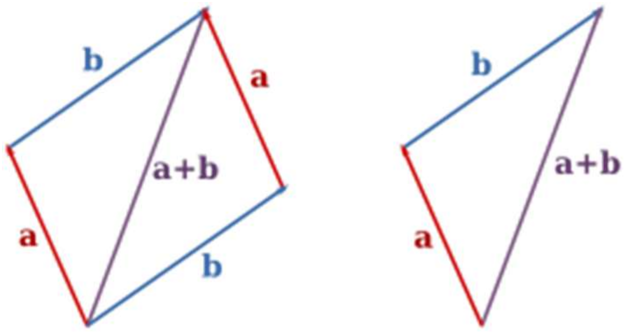
- The sum of **a** and **b** is:

$$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{e_1} + (a_2 + b_2)\mathbf{e_2} + (a_3 + b_3)\mathbf{e_3}.$$

- The addition may be represented graphically by placing the start of the arrow **b** at the tip of the arrow **a**, and then drawing an arrow from the start of **a** to the tip of **b**.

Vector Addition and Subtraction (2)

- The new arrow drawn represents the **vector a** + **b**, as illustrated below:



- This addition method is sometimes called the ***parallelogram rule*** because **a** and **b** form the sides of a parallelogram (a quadrilateral with two pairs of **parallel** sides) and **a** + **b** is one of the diagonals.

- If **a** and **b** are bound **vectors** that have the same base **point**, it will also be the base point of **a** + **b**.

Vector Addition and Subtraction (3)

- This addition method is sometimes called the *parallelogram rule* because **a** and **b** form the sides of a parallelogram (a quadrilateral with two pairs of **parallel** sides) and **a** + **b** is one of the diagonals.

- If **a** and **b** are bound **vectors** that have the same base **point**, it will also be the base point of **a** + **b**.

- One can check geometrically that **a** + **b** = **b** + **a** and (**a** + **b**) + **c** = **a** + (**b** + **c**).

- The difference of **a** and **b** is

$$\mathbf{a} - \mathbf{b} = (a_1 - b_1)\mathbf{e_1} + (a_2 - b_2)\mathbf{e_2} + (a_3 - b_3)\mathbf{e_3}.$$

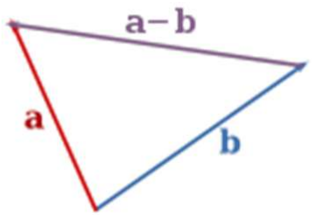
Vector Addition and Subtraction (4)

- The difference of **a** and **b** is:

$$\mathbf{a} - \mathbf{b} = (a_1 - b_1)\mathbf{e_1} + (a_2 - b_2)\mathbf{e_2} + (a_3 - b_3)\mathbf{e_3}.$$

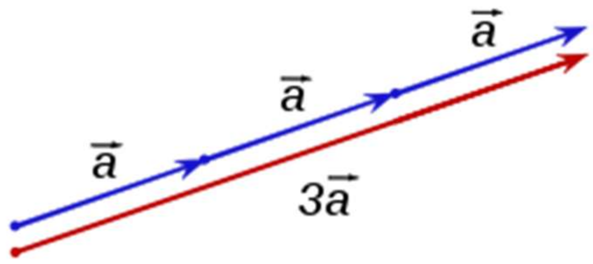
- Subtraction of two **vectors** can be geometrically defined as follows: to subtract **b** from **a**, place the end points of **a** and **b** at the same point, and then draw an arrow from the tip of **b** to the tip of **a**.

- That arrow represents the **vector a** − **b**, as illustrated below:

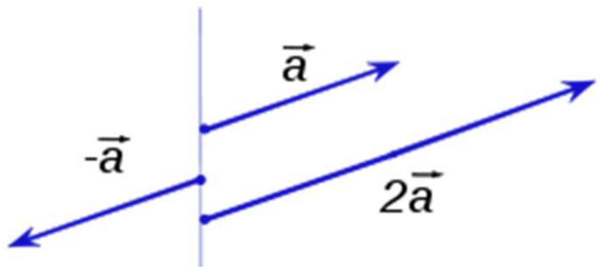


Scalar Multiplication (1)

- Scalar multiplication of a **vector** by a factor of 3 stretches the vector out.



- The scalar multiplications 2**a** and −**a** of a vector **a,**



Scalar Multiplication (2)

- A **vector** may also be multiplied, or re-*scaled*, by a real number *r*.
- In the context of conventional **vector** algebra, these real numbers are often called **scalars** (from *scale*) to distinguish them from **vectors**.
- The operation of multiplying a **vector** by a scalar is called **scalar multiplication**. The resulting **vector** is

$$r\mathbf{a} = (ra_1)\mathbf{e_1} + (ra_2)\mathbf{e_2} + (ra_3)\mathbf{e_3}.$$

Length or Magnitude of a Vector

- The length or magnitude or norm of the **vector a** is denoted by ||**a**|| or, less commonly, |**a**|, which is not to be confused with the absolute value (a scalar "norm").

- The length of the **vector a** can be computed with the **Euclidean** norm, which is a consequence of the Pythagorean theorem since the basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are **orthogonal** unit vectors.

$$\|\mathbf{a}\| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$$

- This happens to be equal to the **square root** of the **dot product** of the **vector** with itself:

$$\|\mathbf{a}\| = \sqrt{\mathbf{a} \cdot \mathbf{a}}.$$

Unit Vector

- The normalization of a **vector a** into a unit **vector â**

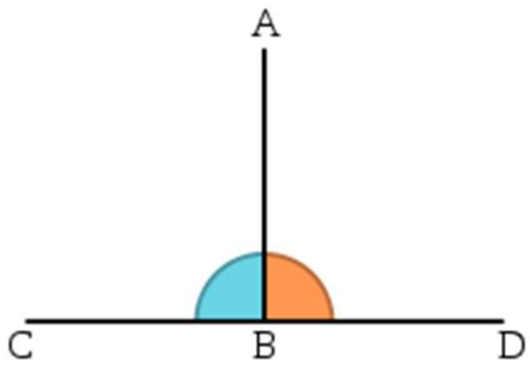
- A **unit vector** is any **vector** with a **length** of one; normally unit **vectors** are used simply to indicate direction. A **vector** of arbitrary **length** can be divided by its **length** to create a unit **vector**. This is known as **normalizing** a **vector**.

- A unit **vector** is often indicated with a hat as in **â**.

- To **normalize** a **vector** **a** = [a_1, a_2, a_3], scale the **vector** by the reciprocal of its length ||**a**||.

Orthogonal Vectors

- Two **vectors** are **orthogonal** if they are **perpendicular**, i.e., they form a right angle.



- The **line segments** AB and CD are **orthogonal** to each other.

Orthonormal Vectors

- Two **vectors** in an inner product space are **orthonormal** if they are **orthogonal** and both of unit length.

- A set of **vectors** form an **orthonormal set** - if all **vectors** in the set are mutually orthogonal and all of **unit length**.

- An **orthonormal set** which forms a basis is called an **orthonormal basis**.

Dot product (1)

- In mathematics, the **dot product** is an algebraic operation that takes two equal-length sequences of numbers (usually coordinate **vectors**) and returns a single number obtained by multiplying corresponding entries and adding up those products.

- The name is derived from the centered dot "·" that is often used to designate this operation; the alternative name **scalar product** emphasizes the scalar (rather than vector) nature of the result.

- The principal use of this product is the **inner product** in a **Euclidean vector** space: when two **vectors** are expressed on an **orthonormal** basis, the **dot product** of their coordinate **vectors** gives their inner product.

- The **dot product** contrasts (in three dimensional space) with the **cross product**, which produces a **vector** as result.

Dot product (2)

- The **dot product** of two vectors **a** and **b** - sometimes called the inner product, or, since its result is a scalar, the scalar product -) is denoted by **a** · **b** and is defined as:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

- Where ϑ is the measure of the angle between **a** and **b** (see trigonometric function for an explanation of cosine).

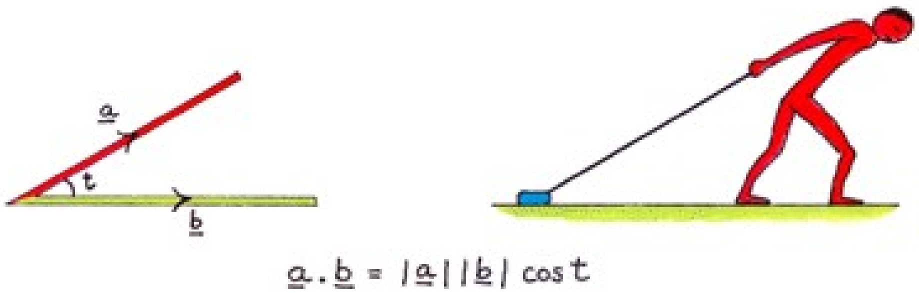
- Geometrically, this means that **a** and **b** are drawn with a common start point and then the length of **a** is multiplied with the length of that component of **b** that points in the same direction as **a**.

- The **dot product** can also be defined as the sum of the products of the components of each **vector** as:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

Dot product (3)

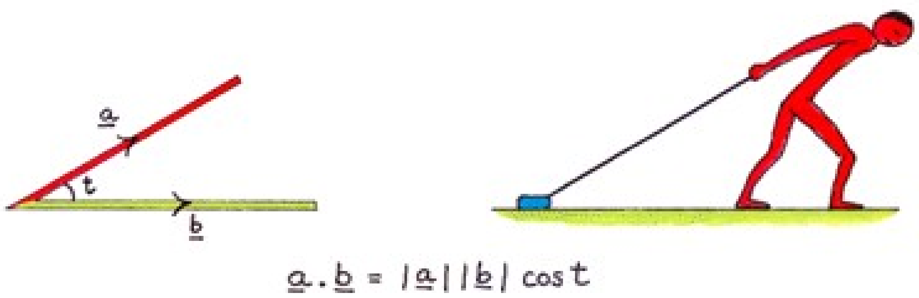
- The picture below gives its definition for two **vectors a** and **b** and also shows a very simple and practical application.



- The man is pulling the block with a constant force **a** so that it moves along the horizontal ground.

Dot product (4)

- The work done in moving the block through a distance **b** is then given by the distance moved through multiplied by the magnitude of the component of the force in the direction of motion.

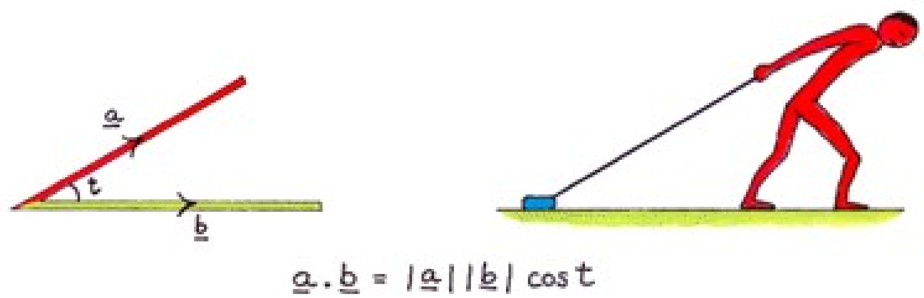


- So we define the scalar or **dot product** as:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

Dot product (5)

- Where t is the angle between **a** and **b** when they are placed tail to tail.



- To use the least amount of force possible, we would need to pull horizontally, so that we are pulling in the same direction as we want the object to move.

- Then we would have t=0 and cos t=1 so that:

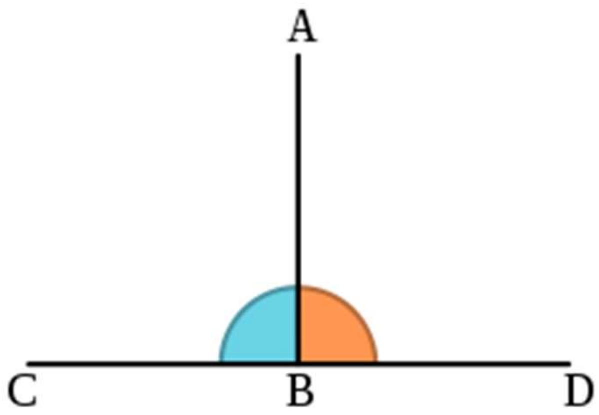
  Work done = **a.b** = |**a**||**b**| = magnitude of the force x distance moved in the direction of the force.

Null Vector

- The **null vector** (or **zero vector**) is the vector with length zero.
- Written out in coordinates, the vector is (0,0,0), and it is commonly denoted , or **0**, or simply 0.
- Unlike any other **vector**, it does not have a direction, and cannot be normalized (that is, there is no unit **vector** which is a multiple of the null **vector**).
- The sum of the null vector with any vector **a** is **a** (that is, **0**+**a**=**a**).

Perpendicular

- In geometry, two lines or **planes** (or a **line** and a **plane**), are considered **perpendicular** (or **orthogonal**) to each other if they form congruent adjacent angles (a T-shape).



- The **segment** AB is **perpendicular** to the **segment** CD, because the two angles it creates (indicated in orange and blue, respectively) are each 90 degrees.

Cross Product (1)

- The **cross product** (also called the **vector** product or outer product) is only meaningful in three (or seven) dimensions.

- The **cross product** differs from the **dot product** primarily in that the result of the **cross product** of two **vectors** is a **vector**.

- The **cross product**, denoted **a** × **b**, is a **vector** perpendicular to both **a** and **b** and is defined as:

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \, \mathbf{n}$$

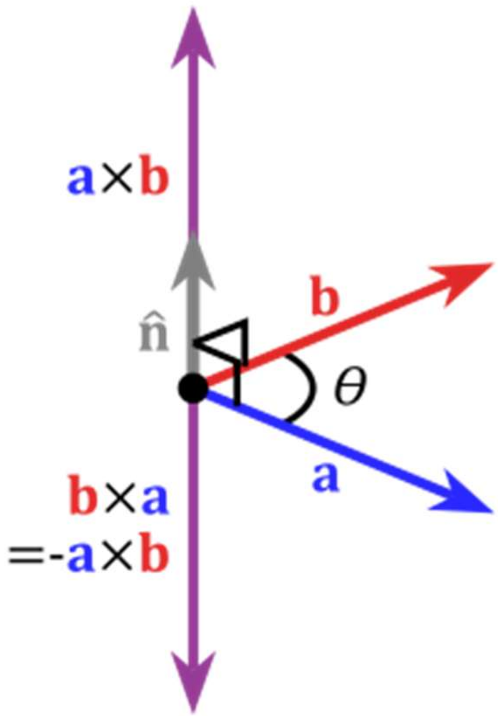
- Where ϑ is the measure of the angle between **a** and **b**, and **n** is a unit **vector** perpendicular to both **a** and **b** which completes a right-handed system.

Cross product (2)

- The right-handedness constraint is necessary because there exist *two* unit **vectors** that are perpendicular to both **a** and **b**, namely, **n** and (–**n**).

- The **cross product a × b** is defined so that **a**, **b**, and **a** × **b** also becomes a right-handed system (but note that **a** and **b** are not necessarily orthogonal). This is the right-hand rule.

- The length of **a** × **b** can be interpreted as the area of the parallelogram having **a** and **b** as sides.



Cross product (3)

- The **cross product** can be written as:

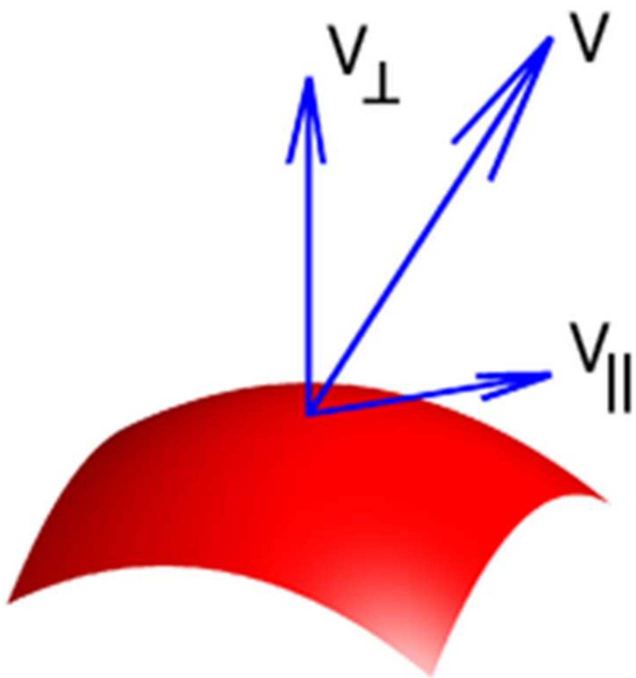
$$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{e}_1 + (a_3b_1 - a_1b_3)\mathbf{e}_2 + (a_1b_2 - a_2b_1)\mathbf{e}_3.$$

Putting it all together (1)

- Given a **vector** at a **point** on a **curve**, that **vector** can be decomposed uniquely as a sum of two **vectors**, one **tangent** to the curve, called the **tangential component** of the **vector**, and another one **perpendicular** to the curve, called the **normal component** of the **vector**.

- Similarly a **vector** at a **point** on a **surface** can be broken down the same way.

- The diagram to the right shows the **Tangential** and **normal components** of a **vector** to a **surface**.



Putting it all together (2)

- More formally, let *S* be a **surface**, and *x* be a **point** on the **surface**.

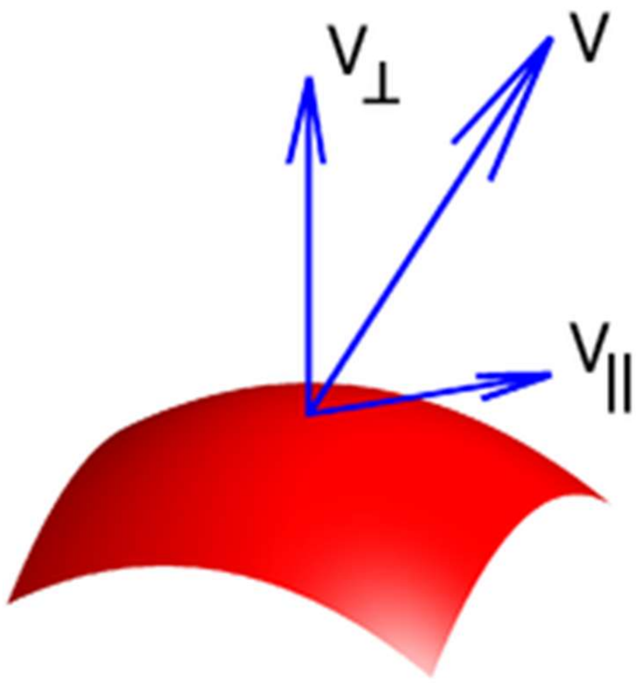
- Let be a **vector** at *x*.

- Then one can write uniquely as a sum:

$$\mathbf{v} = \mathbf{v}_{\|} + \mathbf{v}_{\perp}$$

- Where the first **vector** in the sum is the **tangential component** and the second one is the **normal component**.

- It follows immediately that these two **vectors** are **perpendicular** to each other.



Putting it all together (3)

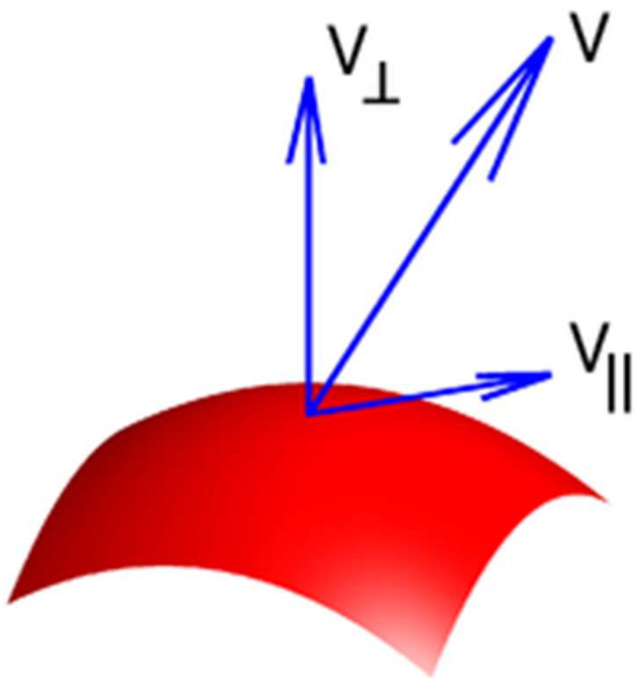
- To calculate the **tangential** and **normal components**, consider a unit normal to the surface, that is, a unit **vector** perpendicular to *S* at *x*. Then,

$$\mathbf{v}_\perp = (\mathbf{v} \cdot \hat{n})\hat{n}$$

- And thus,

$$\mathbf{v}_\parallel = \mathbf{v} - \mathbf{v}_\perp$$

- Where "." denotes the **dot product**.



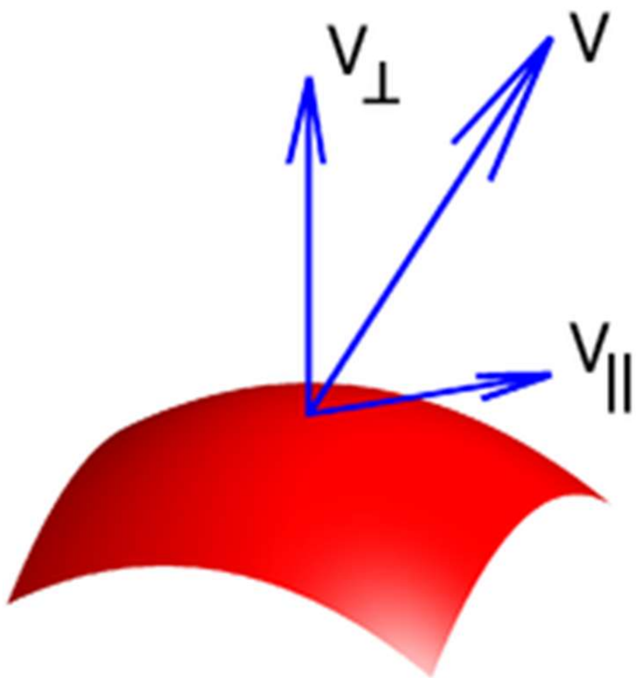
Putting it all together (4)

- Another formula for the **tangential component** is:

$$\mathbf{v}_{\parallel} = -\hat{n} \times (\hat{n} \times \mathbf{v}),$$

- Where “X" denotes the **cross product**.

- Note that these formulas do not depend on the particular unit normal used (there exist two unit normals to any surface at a given **point**, pointing in opposite directions, so one of the unit normals is the negative of the other one).

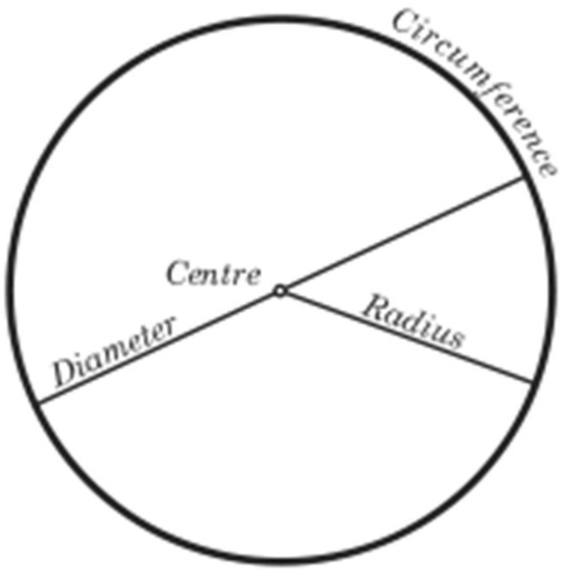


Parametric Curves

Circle (1)

- A **circle** is a simple shape of **Euclidean geometry** consisting of those **points** in a **plane** which are **equidistant** from a given **point** called the **centre**.

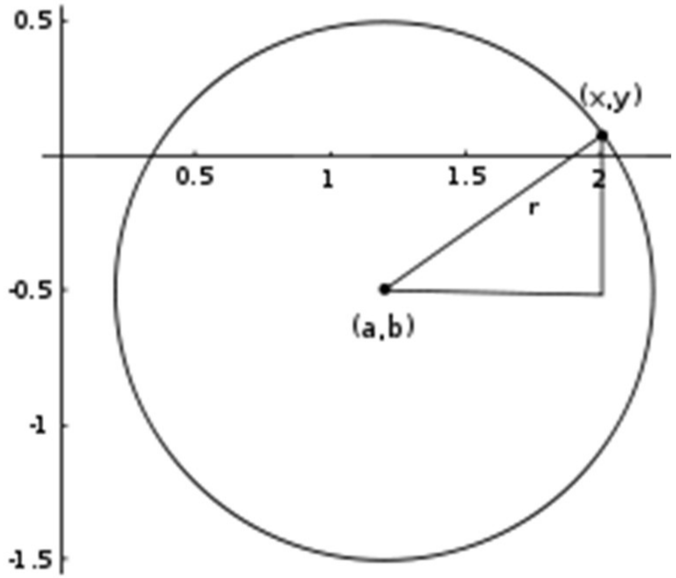
- The common distance of the **points** of a **circle** from its **centre** is called its **radius**.



Circle (2)

- In an *x*-*y* **Cartesian coordinate** system, the **circle** with **centre** (*a*, *b*) and **radius** *r* is the set of all **points** (*x*, *y*) such that:

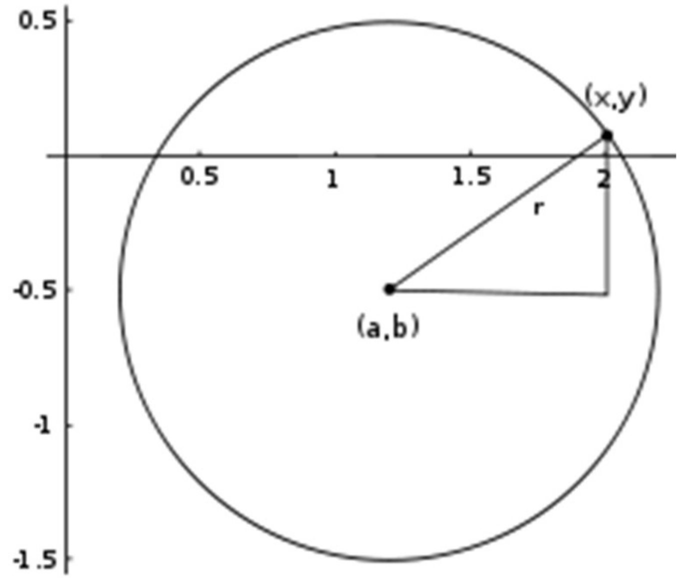
$$(x-a)^2 + (y-b)^2 = r^2.$$



Circle of radius *r* = 1,
centre (*a*, *b*) = (1.2, -0.5)

Circle (3)

- This equation of the **circle** follows from the Pythagorean theorem applied to any point on the **circle**: as shown in the diagram to the right, the **radius** is the hypotenuse of a right-angled triangle whose other sides are of length $x - a$ and $y - b$.



Circle of radius $r = 1$,
centre $(a, b) = (1.2, -0.5)$

Circle (4)

- If the **circle** is centred at the **origin** (0, 0), then the equation simplifies to:

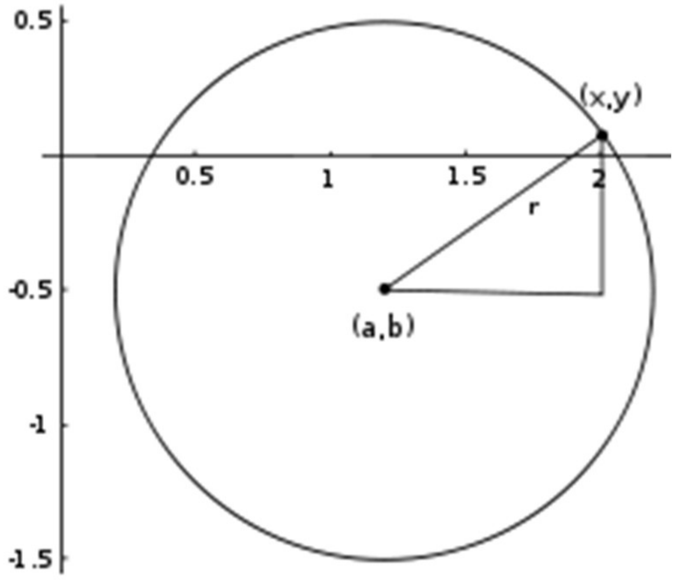
$$x^2 + y^2 = r^2.$$

- The equation can be written in parametric form using the trigonometric functions sine and cosine as:

$$x = a + r\,\cos t,$$

$$y = b + r\,\sin t$$

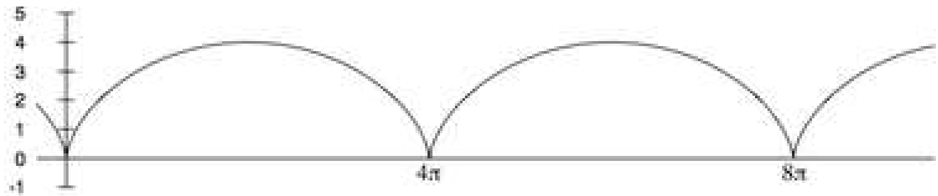
- Where t is a parametric variable, interpreted geometrically as the angle that the ray from the **origin** to (x, y) makes with the x-axis.



Circle of radius r = 1,
centre (a, b) = (1.2, -0.5)

Cycloid (1)

- A **cycloid** is the curve defined by the path of a **point** on the edge of circular wheel as the wheel rolls along a straight **line**. It is an example of a roulette, a **curve** generated by a curve rolling on another **curve**.



A **cycloid** generated by a circle of radius *r* =2

Cycloid (2)

- The cycloid through the origin, generated by a **circle** of **radius** *r*, consists of the **points** (*x*, *y*), with:

$$x = r(t - \sin t)$$
$$y = r(1 - \cos t)$$

- Where *t* is a real parameter, corresponding to the angle through which the rolling **circle** has rotated, measured in radians. For given *t*, the **circle's centre** lies at *x* = *rt*, *y* = *r*.

- Solving for *t* and replacing, the **Cartesian** equation would be:

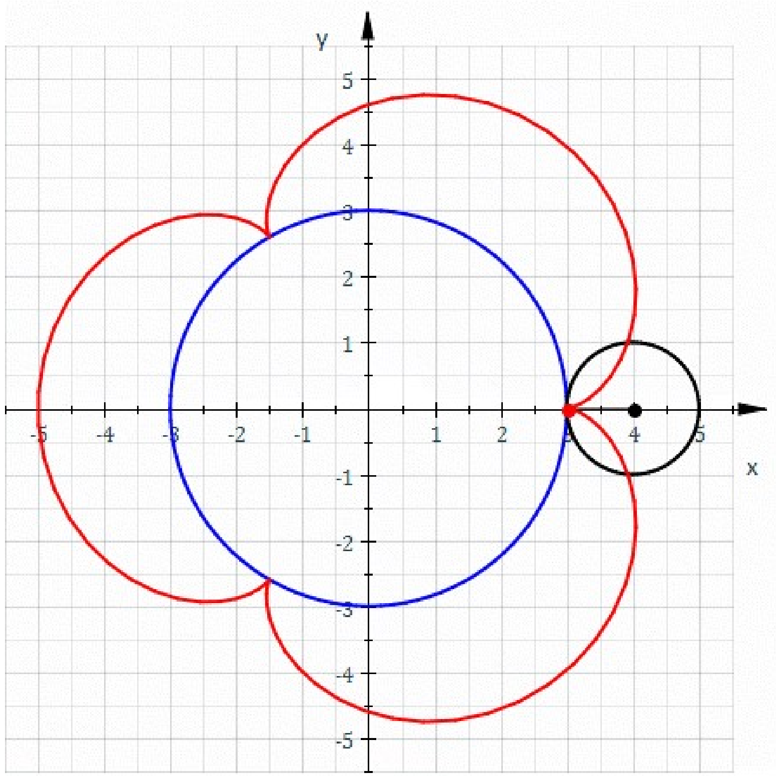
$$x = r\cos^{-1}\left(1 - \frac{y}{r}\right) - \sqrt{y(2r - y)}$$

- The first arch of the **cycloid** consists of points such that:

$$0 \le t \le 2\pi.$$

Epicycloid (1)

- An **epicycloid** is a plane **curve** produced by tracing the path of a chosen **point** of a **circle** — called an *epicycle* — which rolls without slipping around a fixed **circle**. It is a particular kind of roulette.



The red curve is an epicycloid traced as the small **circle** (radius $r = 1$) rolls around the outside of the large **circle** (radius $R = 3$).

Epicycloid (2)

- If the smaller **circle** has radius *r*, and the larger circle has radius *R* = *kr*, then the parametric equations for the curve can be given by either:

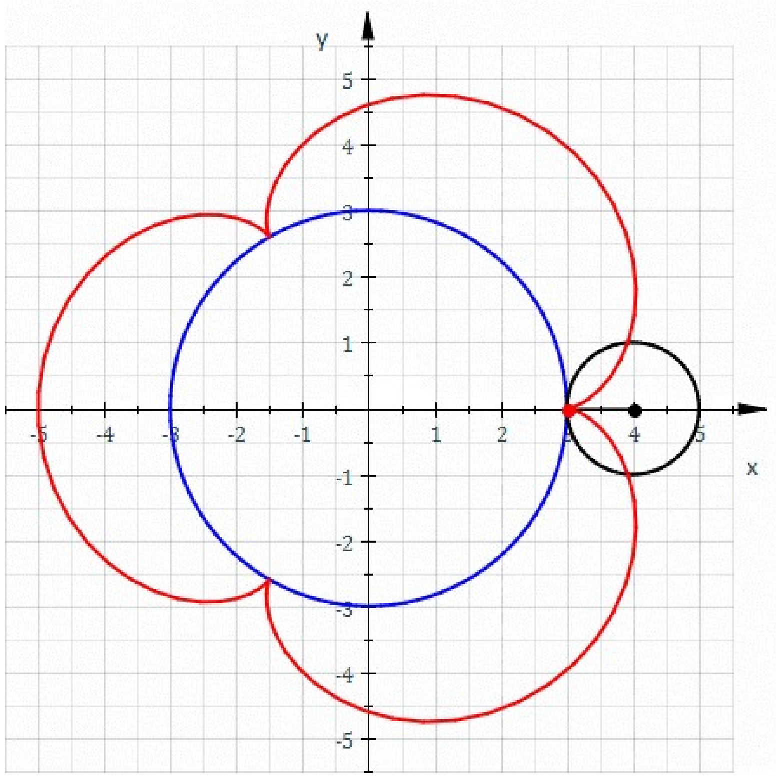
$$x(\theta) = (R+r)\cos\theta - r\cos\left(\frac{R+r}{r}\theta\right)$$

$$y(\theta) = (R+r)\sin\theta - r\sin\left(\frac{R+r}{r}\theta\right),$$

- or:

$$x(\theta) = r(k+1)\cos\theta - r\cos\left((k+1)\theta\right)$$

$$y(\theta) = r(k+1)\sin\theta - r\sin\left((k+1)\theta\right).$$



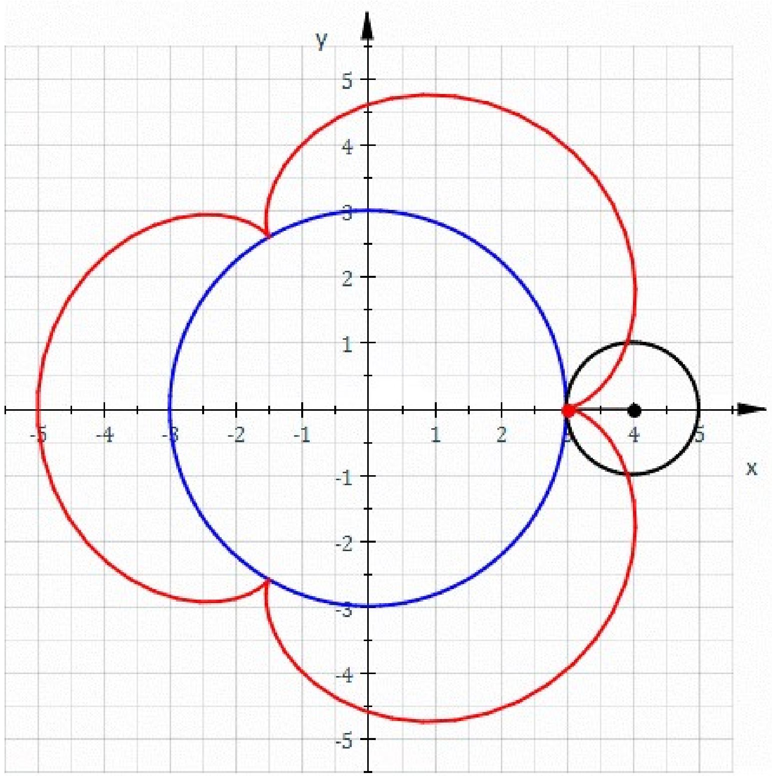
The red curve is an epicycloid traced as the small **circle** (radius *r* = 1) rolls around the outside of the large **circle** (radius *R* = 3).

Epicycloid (3)

- If k is an integer, then the **curve** is closed, and has k cusps (i.e., sharp corners, where the **curve** is not differentiable).

- If k is a rational number, say $k=p/q$ expressed in simplest terms, then the **curve** has p cusps.

- If k is an irrational number, then the **curve** never closes, and forms a dense subset of the space between the larger **circle** and a **circle** of radius $R + 2r$.



The red curve is an epicycloid traced as the small **circle** (radius $r = 1$) rolls around the outside of the large **circle** (radius $R = 3$).

Required For Next Week

Required - reading and work to be undertaken for next week....

So:

- Skim Lengyl Chapters 0 – 2 & READ Chapter 3 on Matrices
- Skim through Dunn & Parberry Chapters 1 to 6 & READ Chapter 7 on Matrices
- If you have already not done so – then compile and run RTVS_Lite
- Next week:
  - Wednesday: Matrices & Introduction to Houdini
  - Thursday: Houdini lecture by SideFX

Course Lecture Notes, Resources and Books

Course materials are hosted – temporarily – on:

- http://www.gjedwards.com/client/msc2012/
  - 11_10_06_Lecture_01_Introduction.zip
  - 11_10_13_Lecture_02_Geometry
  - Resources/
    - Book
      - CurvesSurfGfx-Salomon-2005
      - ELengyel_Mathematics_for_3D_Game_Programming_and_Computer_Graphics
      - F-Dunn-I-Parberry-3DMathPrimerForGraphicsAndGameDevelopment
      - The Algorithmic Beauty of Plants
    - RTVS_Lite.zip

Change of schedule next week

- **Wednesday 17th:**
  - From 11:00 to 13:00:
    - Fundamentals
  - From 13:30 to 17:30:
    - Introduction to Houdini
    - Programming for Vectors using C++ Operator Overloading
    - Introduction to Matrices

- **Thursday 18th:**
  - From 13:30 to 15:30 (and beyond?):
    - Houdini lecture by SideFX

End